A STUDY OF THE FORMAL ARCHITECTURAL-SCULPTURAL CHARACTERISTICS OF EL TAJIN

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1. Introduction

Located on the coast of the Gulf of Mexico and comprising an area of 144 hectares of architectonic buildings, El Tajin was the pre-Hispanic ceremonial center with the highest level of development during the late Classic period, between the years 600-900 [1].

Since its beginnings, the architecture-sculpture of El Tajín[1] has stood out for its use of formal, overlapping elements on its buildings which highlight the importance of the rituals to which they refer, be they of life or of death [2].

The ritual which stands out the most is the Mesoamerican ball game[2]. Until now, twenty ballcourts for practicing this ritual have been found [3]. The survival of this ritual to the present day demonstrates its deep cultural significance [4].

Upon the court walls one can see sculptured, intricately designed bas-reliefs displaying a formal, well-defined pattern organized in a style known as Classical Veracruz, which implies a preoccupation with aesthetics, regularity of shapes, and a flair for ornamentation [5].

The formal architectonic-sculptural structure reveals a geometric and mathematical organization within a parallel system between architectonic thought and mathematical conception [6].

Identifying the geometric pattern implies recognizing the relationships between proportionality, similarity, scale, surroundings and constants [7] pertaining to a visual language related to myths and religion.

The author Bertels, recognized characters and deities belonging to this culture in the bas-reliefs [8]. As such, the recognition of forms is important for the characterization of a culture because it is a way to determine and learn more about its identity. The present work, therefore, attempts to show a structural morphology explained through geometry, complementing the study with the concept of fractality.

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The concept of fractals more rigorously explains the formal constants and composition that give unity to a work of art, while another fractal dimension is presented through formal auto-similarity [9].

In Meso-American architecture, one can observe the formal superimposition of repetition in the different scales of the patterns generated by the building's structure.

As such, the results offer a new focus on the recognition of art forms in which one finds thought and cosmovision materialized in the culture of *El Tajín*, a UNESCO world heritage site.

2. Method

To begin study of the panels' geometric patterns, the *El Tajín* Sculpture Catalogue by Patricia Castillo [10] was used as a reference. From this, high-quality photostatic B/W copies of the sculptural panels' linear drawings of the *South Ballcourt* were obtained to better visualize them.

For this study, the Northwest Panel and the South-Central Panel of the *South Ballcourt* were chosen, as they are considered to be the most important of the site, mainly due to the profusion of images contained in the bas-reliefs of their panels.

Experimental grids were carried out on photostatic copies of the sculptural panels' linear drawings to identify an organization of regular geometric patterns, employing manual instruments including a precision compass, squares and rulers.

This method has already been applied to other buildings in *El Tajin* [11].

Once a regular pattern analogy was identified using the square/rectangle, golden rectangle (RA/Φ), square root rectangle (RR^2) and cube root (RR^3) relationships, the obtained pattern was drawn describing the method in the following way, employing letters to identify geometric reference points.

2.1. Northwest panel (South ballcourt)

Figure 1 - GR-General Rectangle: ABCD composed of RR^2 with CE radius and C-center, an arc is drawn at point D.

EBFD delimits the area containing the skeleton character with large headdress.

RR^2: with DG radius and D-centre, an arc is drawn at point C, in which the straight line GH marks the central axis of the ball player character with headdress.

RR^2: with NG radius and center in N, an arc is drawn at point J, marking the width of the panel's upper frieze.

Its golden section Φ with the straight lines KK', MM' is obtained from the straight line JC. The faces of all the characters in the panel are found lined up in this area.

The golden proportion Φ of the straight line JQ is point P. The straight line PP' marks the central axis of the anthropromorphic human/canine form.

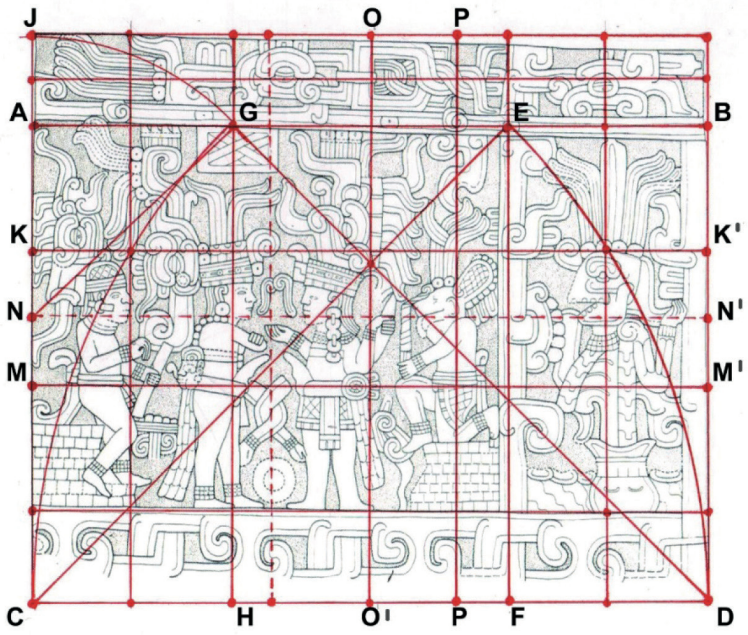

Figure 1. Geometric pattern, Northwest panel, South Ballcourt. El Tajín,

2.2. South central panel, south ballcourt

Figure 2 - From the GR-General Rectangle: ABCD, 2 RR^3 overlapping, with HJ radius and H-center an arc is drawn at point C and with radius GK and G-center an arc is drawn at point D.

The straight line AC and the straight line GH are divided into four equal parts marking important points within the composition of the panel.

The width of the upper frieze is obtained with RA; with PQ radius and P-center an arc is drawn at point R and and that of the bottom frieze with RR^2; with ST radius and S-center an arc is drawn at point U.

The RR^3: with HJ radius and H-center an arc is drawn at point C and with GK radius and G-center an arc is drawn at point D; determine the width of the decorative outer vertical strips.

The RA (Φ): with radius O'E and center in O' an arc is drawn at point V and with radius O'F with center in O' an arc is drawn at point W; determine the vertical strips followed by the outer strips.

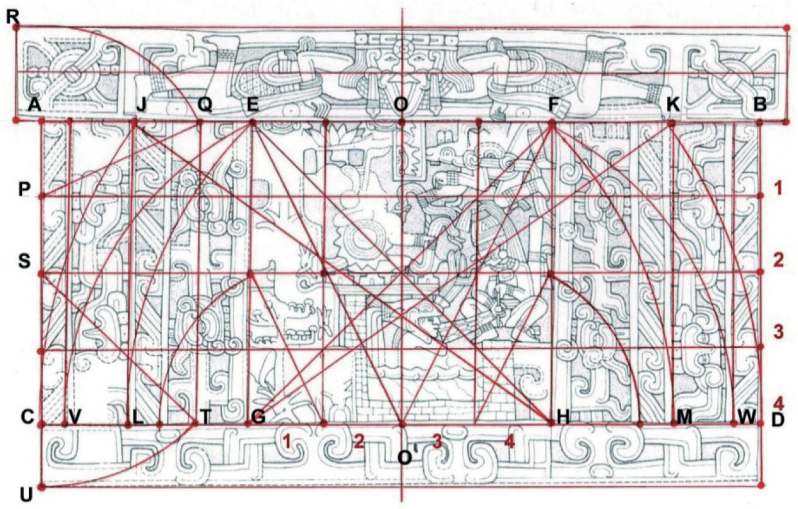

Figure 2. Geometric pattern, south central panel/south ballcourt, El Tajín, México.

The RR²: with HE radius and H-center an arc is drawn at point L and with radius GF and G-center an arc is drawn at point M; are the width of the next strips ordered from outside to inside.

The fractal dimension D was determined by making use of the ImageJ v1.46 program [12] through the relationship [13]:

$$D = \lim_{r \to 0} \frac{\ln N_0(r)}{h\left(1/r\right)}$$

Where (r) is the size of the N sites in which the image is divided and N_0 is the number of sites in which the presence of precipitates is observed.

If one takes into consideration that D constitutes a measure of the quantity of an occupied space Φ which is found on a longitudinal plane, characteristic L:

$$\Phi \equiv L^D$$

it is to be expected that the value of D increases with the amount of precipitate formed.

Fractal geometry has been used to determine architectonic characteristics [14]. In addition to studies of roughness [15], it has also been applied to paintings.

Different methodologies have been presented, with computer simulation as a first option, and size effects and theoretical analysis among others [16], using building scales that can be studied at the microscopic level.

To determine fractality, photographs were taken of the panels. The surface area of the images was calculated using the fractal dimension method by LCD screen and the ImageJ v1.4g program in which each color image is converted to an 8-bit image. From this, a binary image of the pattern showing the exterior borders of the surface area was obtained. Thus, vertical and horizontal fractal dimensions of the images, as well as their texture, were acquired (Figures 3, 4 and 5).

Similar methods were reported by other authors where box-counting fractal dimension is used to characterize image and determine relationship in lines that compounds itself [17-18].

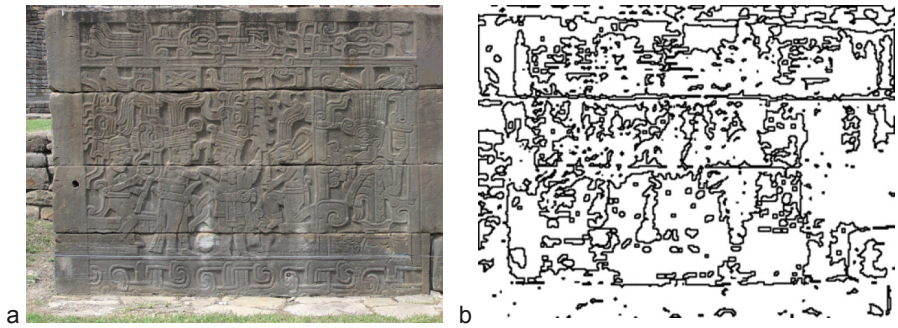

Figure 3. Photograph of the northwest panel; a) original image; b) binary image.

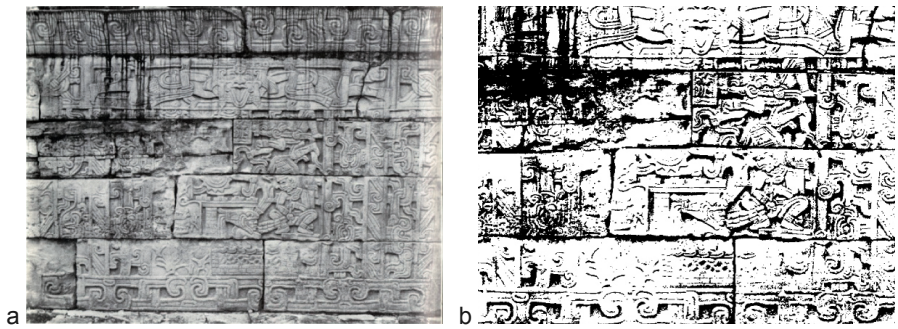

Figure 4. Photograph of the south-central panel before restoration; a) original image; b) binary image.

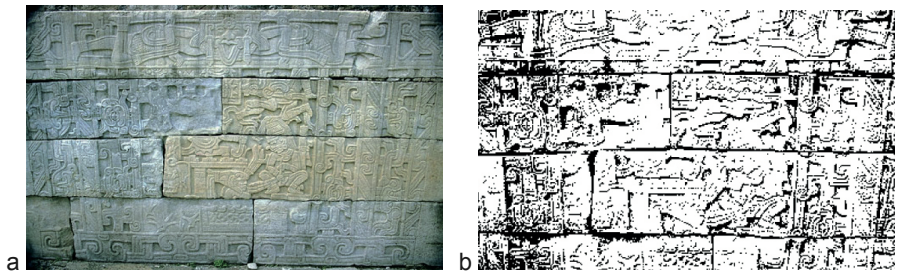

Figure 5. Photograph of the south-central panel after restoration; a) original image, b) binary image.

3. Discussion

General observations regarding the geometric composition of the panels.

The reticular rectangular structure that predominates in the design of the panels adjust appropriately to the general model architecture of ballcourts; the basreliefs are sculpted in the walls, invariably located at the foot of the court, at the so-called benches, which are the closest for the participants in the ritual ballgame (the players). One may deduce that their design, proportion, morphology and semiotics were meant to establish visual contact directed toward a specific objective in the first instance, since in addition to the players, other characters intervened including priests, high-ranking persons, etc.

The interlocking pattern characteristic of the Classic Veracruz style found in the panels, are usually located on the vertical and horizontal strips, although they also appear in the friezes as a border in the corners, intermixed with other motifs, etc. The visual technique utilized in the strips is the rhythmic repetition of motifs interlocked in horizontal and vertical sequences, occupying an important space in the general visual field of the panels.

The *volutas* and interlocking are the unifying graphic motif of the panels' composition due to its insistent presence in all the friezes, vertical strips, clothing and headdresses of the characters. In the semiotics field, they function as basic signs or lexemes belonging to an ancient codifying system as proposed by Pascual Soto. This researcher has studied the morphosyntactic structure by identifying the formal regularities of the signs and their combinatorial order, as well as their semantic value [19].

Within the compositional structure, the upper frieze leads the panel scenes, in which-can be observed the faces of deities and mythical signs.

There is a tendency to orient the composition toward the center of the square or rectangle, so that our vision is drawn to the specific scenes that occur at that point. Within the complexity and dynamism of the images, order and balance is achieved between the horizontal and vertical axes of the composite skeleton. The geometric pattern found in the panels are: RA (Φ), RR^2, RR^3.

The motif repeated in all the panels, as much in the North Ballcourt as in the South Ballcourt, is the eye with *volutas,* highlighted in Figure 6.

As for the position it occupies within the panels, it is visible:

- in the upper frieze
- as a base in the ascending/descending or east/west scenes
- as a part of the character's headdress
- in rhythmic repetition within the vertical strips.

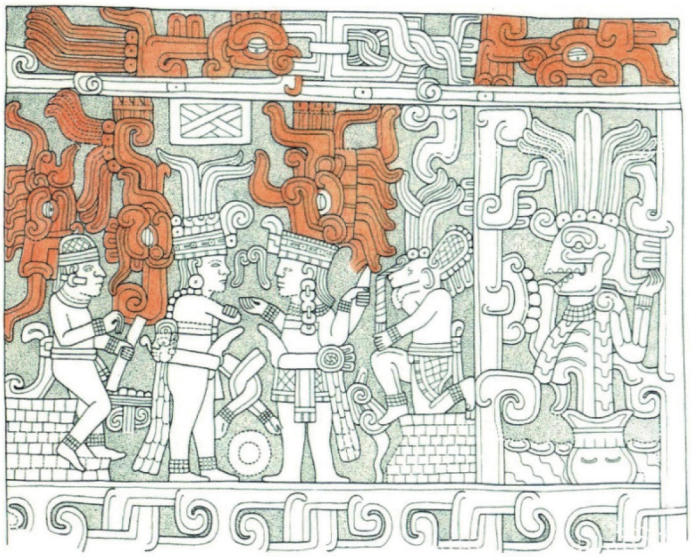

Figure 6. Northwest panel /south ballcourt in which 5 eyes with volutas are highlighted

The different graphic representation variables are:

- with stylized feathers as a border, or in the characters' ritual costume adornment
- with *volutas* as a border.

Another constant motif is that of the *ollín*[3], meaning movement, that normally appears in the characters' costumes; it can also be observed in the friezes and at the base of the panel scenes (Figure 7).

- In the clothing: in the waistband and in the bags of *copalli*[4]

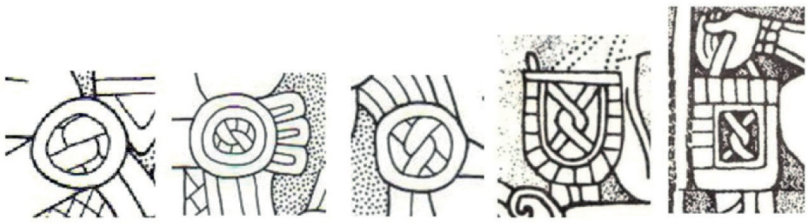

Figure 7. Clothing details from the south ballcourt panel

• In the frieze (Figure 8).

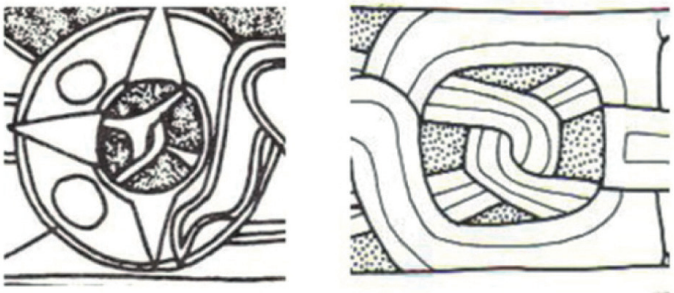

Figure 8. Details in the frieze of the south ballcourt panel

• At the bottom of the panels (Figure 9).

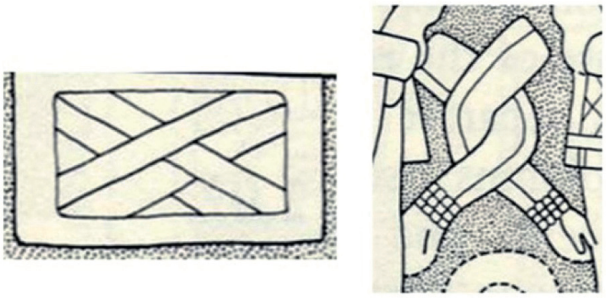

Figure 9. From the south ballcourt panel

The anthropomorphic characters are:
• human/bird, human/dog, human/rabbit according to Piña Chan (Figure 10).

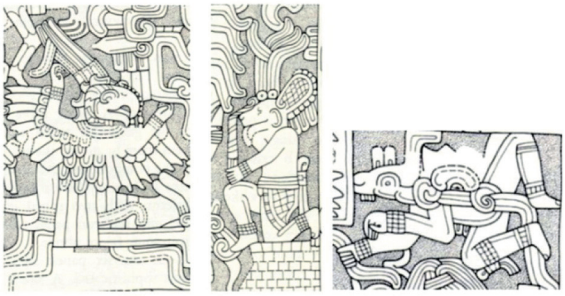

Figure 10. From the south ballcourt panel

The flora and fauna represented include the maguey plant, turtle, stylized serpents with precious feather adornments and an eye with *volutas* for eyelids (Figure 11).

Figure 11. Maguey plant, turtle, interlocked serpents

The description of the naturalist and symbolic elements composing the panels was also facilitated by the reticular organization of their composite geometry, which revealed a formal order responding to a particular cosmic worldview of their ritual and religious thought.

Fractal proportionality presents the following characteristics:

• Geometric composition and dimensioning linked to sacred and astronomical numerology provided ancient American cultures with a highly rigorous scientific vision [20].

• Mandelbrot established that a fractal is a special kind of symmetry or invariance, that relates a whole to its parts and the whole can be broken down into parts that evoke the whole [21].

• The harmonic proportion systems of pre-Hispanic images lead us to fractal geometry. The interpretation is a result of the Mandelbrot method described in the previous paragraphs.

• Non-Euclidian geometry, also known as fractal geometry, allows for a broadening of the understanding of the processes of morphogenesis as another way to observe existing reality [22].

• Since a fractal is a geometric construction, as a shape in space, it is possible to show the fractal in the geometric constructions made by man, such as population sites, architecture and art [23].

• The contribution of fractal geometry as a generating instrument gives architecture the power to simultaneously express two seemingly contradictory states: orderly forms and organic complexity [24].

• A fractal organization happens when there is a motif or a regular design that repeats itself as it grows or changes its scale.

• The value of the fractal dimension in a system can vary depending on the dynamic processes taking place and its random nature by which one can describe a system through its morphology. In this work, the morphologies correspond to the images obtained through the previously described experiments.

Table 1 shows the results of fractal dimension analysis for the borders of the reliefs in the studied areas.

Table 1. Fractal dimension results for case-study panels

	OBSERVATION	FRACTAL DIMENSION	ERROR
Northwest panel	Elaborate curvatures can be observed representing the main ball players.	1.612	0.081
Central south panel (before restoration)	The most elaborate curved shapes highlight the larger dimensions and major visual morphology.	1.683	0.045
Central south panel (after restoration)	Defined elements	1.701	0.085

As one can see, the values differ by less than 7%, and considering the margins of error, no significant differences were found among the results.

Formally, the fractal dimension values qualitatively express that the elements studied have similar complexity. Indeed, the set of combined elements present a set of more elaborate curvatures compared to other elements such as the *volutas,* which tend to present linear geometric shapes or simpler geometric combinations.

The previously described fractal dimension is obtained mathematically through a relationship between individual elements with a topographic dimension and the number of times they fit within a figure.

In this sense, the images studied for *El Tajín* show a fractal dimension, that is, complexity in the arrangement of the lines that form the picture.

The study shows the fractal dimension value is reduced in the images with the degraded figures and is increased when those figures are restored. This indicates, therefore, that for the case in question, the mathematical complexity diminishes with time due to the degradation of the figures, which is coherent with what can be observed visually.

On the other hand, in the studies undertaken for the images it was found that the results contained no significant differences, but they did appear in the other experimental cases of different figures studied for the same ceremonial center. This situation was also found in the study by means of quadratic relations in which they had differing geometric dispositions but where an RR2 relationship also prevailed and is a derivation of the golden rectangle.

As such, when the study shown in this work was carried out with pre-Hispanic vestiges, the conserved elements displayed a fractal dimension. Even though in Mesoamerican cultures, the term ‘fractal dimension’ was unknown, this mathematical geometry could have been instinctively applied not only to relate to in a divine sense, but also with the natural use of profound logical reasoning. In this regard, Dehouve affirms that, in Mesoamerica, the plastic forms of fractal nature are generally widespread [23].

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4. Conclusions

Formal resources were placed in function and at the disposition of the ritual concept.

The formal organization of architecture-sculpture in *El Tajín* is a means of identitying its culture. Involving architectonic spatiality in a temporary context is something which we can access only with difficulty because there are no written testimonials, but with multi-disciplinary studies that complement this research, criteria for a better interpretation and characterization of this ancient culture could be achieved.

In composite geometric analysis, the square root rectangle (RR^2) prevails as the most constant pattern. The golden rectangle and the cubic root rectangle are also employed, having the square root as its initial base, which in Mesoamerican cultures, represents the four cardinal points.

Geometric relationships are in accordance with the proportions and scales of the formal elements of which they are composed.

In architectural works, fractal geometry is observed in patterns of formal similarity that are repeated in different scales; the form and identity of the building are understood through the formal interactions of the compositional structure: the whole with its parts.

Fractal patterns may or may not have been used intentionally in *El Tajin*, however, there remains the fact that fractal values were found in its buildings.

Taken together, the formal considerations resulting from the visual semantics and interpretation of the panels carried out in this study, will complement and add to the current knowledge of the cultural heritage of *El Tajin*.

Fractal and Euclidian geometry complement each other to generate new pertinent models for the study of ancient art. As in nature, form is the complement of function. Hence, forms have a reason for being, and it is because of them that the study of formal composition helps bring us to a better understanding and perception of cultural vestiges.

Acknowledgments

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Notes

1 whc.unesco.org/en/list/631/video

2 The Mesoamerican ballgame was a ritual/sport played from 1800 B.C. There were different versions of the ritual/sport in different places during the millennia and today, a newer, more modern version of the game, ulama, is still played in a few places by the indigenous population.

3 According to the *Nahuatl* dictionary, *ollin* means “movement.” For the Aztecs it is the mystical sign of the Lord of Movement called Ehecatl.

4 In pre-Hispanic times, among the Mexicans or Aztecs, it was known as *copalquáhuitl*: “copal tree”, and the resin extracted from it as *copalli*: “incense”. the Mexicans used it for their rituals.

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Summary

El Tajín was an ancient metropolis in which rituals such as the Mesoamerican ball game were carried out, later to be recorded in the sculptural bas-reliefs of its architecture. The study of its morphologies is the recognition of the ways in which an ancient

civilization is expressed, thus contributing to the characterization of a culture whose past belongs to World Heritage. This paper proposes a case-sample analysis of the bas-reliefs in the South Ballcourt based on reticular geometry and fractal dimension analysis. It was found that the geometry of the RA (golden rectangle), RR^2 and RR^3 are prevalent, in addition to the identification of iconographic naturalist and symbolic elements; from the box-counting fractal dimension, it was found that the elements, though of different sizes or composition, show similar complexities, with a value of around 1.7

Riassunto

El Tajín era un'antica metropoli in cui venivano eseguiti rituali come il gioco della palla mesoamericana. Questo gioco è riportato, a livello architettonico, su bassorilievi. Lo studio delle modalità espressive di un'antica civiltà contribuisce alla caratterizzazione di una cultura, il cui passato appartiene al Patrimonio Mondiale. Questo articolo propone un'analisi del caso di studio dei bassorilievi nel South Ballcourt basata sulla geometria reticolare e l'analisi della dimensione frattale. Si è riscontrato che prevalgono la geometria di RA (rettangolo aureo), RR2 e RR3. Oltre all'identificazione dell'iconografia e degli elementi simbolici dei bassorilievi, dalla dimensione frattale di conteggio delle caselle si è scoperto che gli elementi, sebbene di dimensioni o composizione diverse, mostrano complessità simili, con un valore di circa 1,7.